Bayesian Approaches to Carbon Cycle Modelling

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Abstract

Increases in atmospheric CO_2 and other greenhouse gases are responsible for global climate change. Large-scale transfer of carbon from earth's soil to the atmosphere has the potential to globally reduce soil quality and agricultural productivity. Accurate models of soil carbon cycles for forecasting future changes are useful tools to help understand and alleviate these problems. Soil carbon sequestration, has many benefits: helping to reduce the atmospheric concentration of greenhouse gases, improving agricultural productivity, and potential financial gains for farmers. Advanced Bayesian models for a soil carbon cycle are fit using Markov chain Monte Carlo (MCMC) methods to quantify uncertainty in a soil carbon cycle in order to determine how soil carbon sequestration can best be used to reduce greenhouse gas, energy costs and improve soil productivity.

Introduction

Global carbon cycling describes the exchange of carbon between and within four main pools: earth's oceans, its atmosphere, fossil fuels, and its soil. Soils store approximately 4 times more carbon than the atmosphere and 6 times more than earth's vegetation. Therefore it is important that we have reliable statistical approaches for monitoring this fundamentally important carbon pool. The challenge is that the understanding of the sequestration process and the carbon cycle is imperfect. Some research has been conducted into quantifying uncertainty in soil carbon model outputs typically under simulated climate change or via a sensitivity analysis by running models for different sets of parameter values. We develop and evaluate advanced Bayesian methods for: modelling soil carbon sequestration, estimating parameters in a statistically defensible way, and improving the speed of computations.

Materials and Methods

The model of soil carbon cycling for one-pool model which is used by Clifford *et. al.* [3], is:

$$\log(X^i_{C(t)}) = \log(X^i_{C(t-1)}e^{-K} + I^i_{C(t)}) + \eta^i_t, \quad \eta \sim N(0, \sigma^2_\eta);$$
$$X^i_{G(t)} \sim LN(\mu_G + \rho_G(\log(X^i_{G(t-1)}) - \mu_G), \sigma^2_G);$$
$$X^i_{W(t)} \sim LN(\log h + \log(X^i_{G(t)}), \sigma^2_W);$$
$$X^i_{P(t)} \sim LN(\mu_P + \rho_P(\log(X^i_{P(t-1)}) - \mu_P), \sigma^2_P);$$

where $X^i_{C(t)}$, $X^i_{G(t)}$, $X^i_{W(t)}$, and $X^i_{P(t)}$ denote, respectively, the masses of SOC, total grain dry matter, total wheat dry matter, and total pasture dry matter produced in field i at time t. The carbon inputs $I_C(i,t)$ is:

$$I^i_{C(t)} = \begin{cases} c(X^i_{W(t)} - X^i_{G(t)}) + cr_W X^i_{W(t)} & \text{Wheat for Grain} \\ cpX^i_{W(t)} + cr_W X^i_{W(t)} & \text{Wheat for Hay} \\ cX^i_{P(t)} + cr_P X^i_{P(t)} & \text{Pasture} \\ cpX^i_{P(t)} + cr_P X^i_{P(t)} & \text{Pasture for Hay} \\ 0 & \text{Fallow} \end{cases}$$

where c is the carbon content of generic plant material, ρ is the proportion of the crop left above-ground as stubble following harvesting as hay, and r_W and r_P are, respectively, the root-to-shoot ratios for wheat crops and for pasture. Also, the measurement models are shown in the following:

$$Y^i_{C(t)}|X^i_{C(t)} = x^i_{C(t)} \sim LN(log(x^i_{C(t)}), \sigma^2_{\epsilon C});$$
$$Y^i_{G(t)}|X^i_{G(t)} = x^i_{G(t)} \sim LN(log(x^i_{G(t)}), \sigma^2_{\epsilon G});$$
$$Y^i_{W(t)}|X^i_{W(t)} = x^i_{W(t)} \sim LN(log(x^i_{W(t)}), \sigma^2_{\epsilon W});$$
$$Y^i_{P(t)}|X^i_{P(t)} = x^i_{P(t)} \sim LN(log(x^i_{P(t)}), \sigma^2_{\epsilon P}).$$

The Bootstrap Particle Filter (BPF) is an iterative method for carrying out Bayesian inference for dynamic state space models, which can be nonlinear [5].

The Kalman Filter (KF) is an optimal estimator in the sense of minimizing the variance of the estimated states and a recursive filter algorithm for linear state-space models that allows new measurements to be processed as they arrive [2].

Rao-Blackwellised particle filters (RBPFs) are used where the model is a combination of linear and non-linear variables, the linear part can be estimated by (KF) and the non-linear part can be estimated by BPF [4].

The Particle Marginal Metropolis-Hastings (PMMH) algorithm is a MCMC method for generating a sequence of random samples from a probability distribution from which direct sampling is difficult. This sequence can be utilized to approximate the distribution or to estimate an expectation with respect to the posterior. This scheme is used in the case of non-linear and non-Gaussian state-space model that the marginal likelihood required in the acceptance ratio is often difficult to compute [1] .

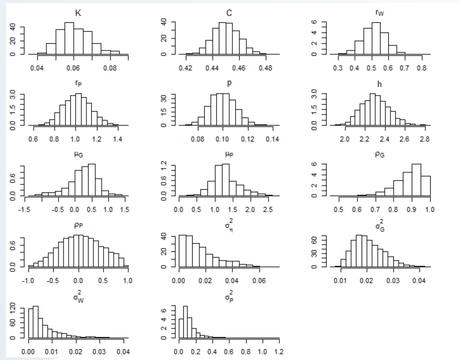
Mathematical Section

The state-space model uses indirectly observable variables known as state (or latent) variables and observable measurement variables to describe a system. These state variables cannot be measured directly and are estimated through measured data [6]. The model is:

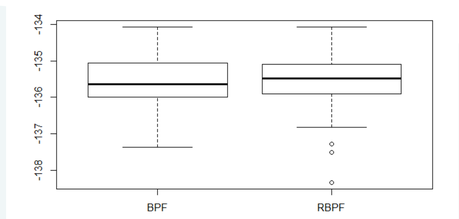
$$X_t = f(X_{t-1}) + B_t u_t + \epsilon$$
$$Y_t = g(X_t) + \nu;$$

where $\epsilon \sim N(\mu, \sigma^2_X)$ and $\nu \sim N(\lambda, \sigma^2_Y)$ are state and measurement noise components, respectively.

Results



The histogram for estimated parameters base on RBPF



The box plots of the estimated log likelihood using BPF and RBPF

Method	Number of particles	Elapsed time (second)
BPF	550	8.53
RBPF	**55**	**4.9**

The number of particles and the elapsed time of computations based on RBPF and BPF algorithms.

Conclusions

- RBPF perform faster than BPF and needs fewer particles.
- RBPF has more precision than BPF since it computes the exact log-likelihoods of linear variables.

Forthcoming Research

Other advanced Bayesian methods can be used on the model for improving the precision of the estimation and increase the speed of the computation. Also, comparing one-pool model and multi-pool model for modeling soil carbon cycling is of interest.

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